PreCalculus/ PreCalculus

Unit 1: Stepping into Precal

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and **P.1G** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

P.2A use the composition of two functions to model and solve real-world problems;

P.2B demonstrate that function composition is not always commutative;

P.2C represent a given function as a composite function of two or more functions;

P.2D describe symmetry of graphs of even and odd functions;

P.2E determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

P.2F graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

P.21 determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

P.2J analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

P.2L determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities; **P.2M** describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities.

- I can describe the symmetry of graphs of even and odd functions using precise mathematical language and justifying if a function is even or odd using multiple representations.
- I can describe end behavior of functions using precise mathematical language in mathematical and real-world problems. I can determine various types of discontinuities and describe the left-sided behavior and the right-sided behavior of the graph of a function around those discontinuities using precise mathematical language.
- I can use and represent the composition of two functions to model and solve real-world problems. I can demonstrate that function composition is not always commutative and that a function can be written as a composition of two or more functions.
- I can determine an inverse function, when it exists, and represent the inverse using multiple representations.
- I can graph piecewise and step functions by selecting appropriate tools and techniques. I can describe key features of piecewise defined functions and step functions using precise mathematical language.

Unit 2: How Many are Rational?

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

P.2D describe symmetry of graphs of even and odd functions;

P.2F graph-exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

P.2G graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including af(x), f(x) + d, f(x - c), f(bx) for specific values of a, b, c, and d, in mathematical and real-world problems;

P.21 determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

P.2J analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

P.2K analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

P.2L determine various types of discontinuities in the interval ($-\infty$, ∞) as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

P.2M describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities; and
 P.2N analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems.

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

P.5J solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems;

P.5K solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems; and

P.5L solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems.

- I can graph polynomial and power functions and their transformations by selecting appropriate tools and techniques. I can write equations and describe key features of polynomial and power functions using precise mathematical language.
- I can solve polynomial equations in mathematical and real-world problems.
- I can solve polynomial inequalities in mathematical and real-world problems.
- I can analyze situations modeled by polynomial and power functions to solve real-world problems.
- I can graph rational functions and their transformations by selecting appropriate tools and techniques. I can write equations and describe key features of rational functions using precise mathematical language.
- I can solve rational inequalities in mathematical and real-world problems.
- I can analyze situations modeled by rational functions to solve real-world problems.

Unit 3: Power Up, Log On

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to: P.2E determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

P.2F graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

P.2G graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power-functions and their transformations, including af(x), f(x) + d, f(x - c), f(bx) for specific values of a, b, c, and d, in mathematical and real-world problems;

P.21 determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

P.2J analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power-functions, using infinity notation to communicate this characteristic in mathematical and real-world problems; and

P.2N analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems.

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to: P.5G use the properties of logarithms to evaluate or transform logarithmic expressions;

P.5H generate and solve logarithmic equations in mathematical and real-world problems; and

P.5I generate and solve exponential equations in mathematical and real-world problems.

- I can graph exponential functions and their transformations by selecting appropriate tools and techniques. I can describe key features of exponential functions using precise mathematical language.
- I can graph logarithmic functions and their transformations by selecting appropriate tools and techniques. I can describe key features of logarithmic functions using precise mathematical language.
- I can generate and solve exponential and logarithmic equations in mathematical and real-world problems.
- I can analyze situations modeled by exponential and logarithmic functions to solve real-world problems.
- I can use the properties of logarithms to evaluate or transform logarithmic expressions.

Unit 4: The Unit Circle of Life

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and **P.1G** display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

P.2P determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

P.4B describe the relationship between degree and radian measure on the unit circle;

P.4C represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;

P.4D represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;

P.4E determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;

P.4F use trigonometry in mathematical and real-world problems, including directional bearing.

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

P.5N generate and solve trigonometric equations in mathematical and real-world problems.

- I can solve problems involving trigonometric ratios, inverse trigonometric functions, and trigonometry in mathematical and real-world problems.
- I can determine angles in standard position and coterminal angles by selecting appropriate tools and techniques. I can represent and describe the relationship between angles in radians or degrees on the unit circle. I can convert between radians and degrees by selecting appropriate tools and techniques.
- I can determine reference angles and use special right triangles and angle quadrant location to determine trigonometric values on the unit circle by selecting appropriate tools and techniques.

Unit 5: SINE of the Times

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to: P.2E determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

P.2F graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

P.2G graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power-functions and their transformations, including af(x), f(x) + d, f(x - c), f(bx) for specific values of a, b, c, and d, in mathematical and real-world problems;

P.2H graph arcsin x and arccos x and describe the limitations on the domain;

P.21 determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

P.20 develop and use a sinusoidal function that models a situation in mathematical and real-world problems.

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

P.4A determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.

- I can explain the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.
- I can graph trigonometric and inverse trigonometric functions by selecting appropriate tools and techniques.
 I can describe key features of trigonometric and inverse trigonometric functions using precise mathematical language.

- I can graph sine and cosine functions and their transformations in mathematical and real-world problems.
- I can determine values of inverse trigonometric functions, when they exist, and represent the relationship between a trigonometric function and its inverse using tables, graphs, and equations.
- I can determine and use a sinusoidal function that models a situation in mathematical and real-world problems.

Unit 6: Positive Identity

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

P.5M use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and

P.5N generate and solve trigonometric equations in mathematical and real-world problems.

- I can simplify and evaluate trigonometric expressions using trigonometric identities. I can explain my reasoning using logical arguments.
- I can solve trigonometric equations in mathematical and real-world problems. I can explain my reasoning using logical arguments and determine reasonableness of solutions.

Unit 7: Find Your Bearings

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

P.4F use trigonometry in mathematical and real-world problems, including directional bearing;

P.4G use the Law of Sines in mathematical and real-world problems;

P.4H use the Law of Cosines in mathematical and real-world problems;

P.4I use vectors to model situations involving magnitude and direction;

P.4J represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and

P.4K apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

- I can use Law of Sines and Law of Cosines to calculate measures in mathematical and real world problems. I can justify my reasoning and determine reasonableness of solutions.
- I can use vectors to model situations involving magnitude and direction. I can use representations to justify my reasoning.
- I can represent and apply vector addition and scalar multiplication in order to solve mathematical and real-world problems. I can justify my reasoning and determine reasonableness of solutions.

Unit 8: Sum It All Up

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

P.5A evaluate finite sums and geometric series, when possible, written in sigma notation;

P.5B represent arithmetic sequences and geometric sequences using recursive formulas;

P.5C calculate the nth term and the nth partial sum of an arithmetic series in mathematical and real-world problems;

P.5D represent arithmetic series and geometric series using sigma notation;

P.5E calculate the nth term of a geometric series, the nth partial sum of a geometric series, and sum of an infinite geometric series when it exists; and

P.5F apply the Binomial Theorem for the expansion of $(a + b)^n$ in powers of a and b for a positive integer n, where a and b are any numbers.

- I can investigate different types of sequences and use sigma notation to represent and calculate sums of series.
- I can represent arithmetic sequences. I can represent and find sums of arithmetic series in mathematical and real-world problems. I can justify my reasoning and determine reasonableness of solutions.
- I can represent geometric sequences. I can represent and find sums of geometric series in mathematical and real-world problems. I can justify my reasoning and determine reasonableness of solutions.
- I can use the Binomial Theorem to efficiently and effectively expand binomials in the form (a + b)ⁿ. I can justify and represent my reasoning.

Unit 9: ConicCon

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

P.3F determine the conic section formed when a plane intersects a double-napped cone;

P.3G make connections between the locus definition of conic sections and their equations in rectangular coordinates;

P.3H use the characteristics of an ellipse to write the equation of an ellipse with center (h, k); and **P.3I** use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k).

- I can determine the conic section formed when a plane intersects a double-napped cone and communicate my mathematical reasoning using multiple representations, including symbols, diagrams, and language as appropriate.
- I can write and represent the equation of an ellipse and a hyperbola using given attributes and justify my thinking.

Unit 10: The Polar Express

Texas Essential Knowledge and Skills (TEKS)

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

P.1A apply mathematics to problems arising in everyday life, society, and the workplace;

P.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

P.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

P.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

P.1E create and use representations to organize, record, and communicate mathematical ideas;

P.1F analyze mathematical relationships to connect and communicate mathematical ideas; and

P.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

P.3A graph a set of parametric equations;

P.3B convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;

P.3C use parametric equations to model and solve mathematical and real-world problems;

P.3D graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates; and

P.3E graph polar equations by plotting points and using technology.

- I can graph parametric equations with and without technology and identify key features to describe mathematical and real-world contexts.
- I can convert between parametric equations and rectangular relations. I can explain my reasoning using logical arguments and determine reasonableness of solutions.
- I can graph points in the polar coordinate system.
- I can convert between rectangular coordinates and polar coordinates.
- I can graph polar equations by plotting points and using technology and identify key features to describe mathematical and real-world contexts.